

**AMENDMENT TO THE SPECIFICATION**

*Page 4--amend the first full paragraph*

[0013] The topology of the actual representation can vary if the model is only detailed up to bus bar level, which may suit off-line studies for Planning Engineers. Yet for operations, the model must reach switching levels. Modeling for other purposes can also be done, as described in U.S. Patent No. 6,202,041 to Tse et al., which discloses a modeling method for small perturbation stability, as well as U.S. Patent No. 6,141,482 6,141,634 to Flint et al., which discloses an AC power line network simulator.

*Page 5--amend the paragraph that carries over to page 6 and add two new paragraphs before the last sentence*

[0021] Several attempts to overcome these difficulties have been undertaken in the past, but with limited success. For example, load flow and state estimators currently used in electrical advanced applications at control centers, represent the state-of-the-art technology: Newton-Raphson Iterative methodology, as well as variants for improving convergence and speed of computation (Fast decoupling, etc.), avoiding triangulation of the Jacobian, as well as new approaches using fuzzy logic and genetic algorithms.

An interesting approach to the load flow equations solutions is the use of the General Homotopy Method proposed by Okumura et al., "A computation of Power System Characteristic by General Homotopy and Investigation of its Stability", in which is defined a family of solutions characterized by a continuous real parameter defining a path. The path must be followed step-by-step using a predictor-corrector methodology involving Newton-Raphson in each step. In order to avoid the inherent Newton-Raphson drawbacks, the step size must be carefully chosen in order not to lose the path to the solution.

Other approximations to the load flow equation solutions include Tolikas et al. "Homotopy Methods for solving Decoupled Power Flow equations" or Guo et al., "The Homotopy

Continuation Method to Approach Voltage Collapse of Electric Power Systems". Any of the approaches results are reliable enough and efficient for industrial use in a real time environment.

The list of references on this matter is not exhaustive but its length indicates that it is a problem yet to be solved to complete satisfaction.

*Page 7--amend the first paragraph*

[0025] 3) Optimal power ~~flow~~ flow for objective functions such as losses minimization through reactive power cycling.

*Page 11--amend the second, third and fourth full paragraphs*

[0044] a. Embed the load flow problem L in a ~~parametric homotopy~~ holomorphic embedding that goes continuously from the ~~[[0-case]]~~ no load case (the case for which there is not load, nor generation, and consequently no flow in the lines) to the problem case;

[0045] b. Develop, in power series, the values of the equation's unknowns in the parameters of the ~~homotopy~~ embedding in a neighborhood of the ~~[[0-case]]~~ no load case value of the parameter; and

[0046] c. Use analytical continuity (by a n-order algebraic approximant) to find the value for the equation's unknowns in the problem case.

*Page 11--amend the sixth full paragraph*

[0048] ~~Figures 1a-1e are schematic representations of convergence regions realized through implementation of a prior art FDNR method on~~ Figure 1 is a representation of a two-bus network.

*Page 14-15--delete the last two paragraphs on page 14 (beginning at line 18) and the first three paragraphs on page 15, and add two new paragraphs beginning at line 17 (page 15)*

This problem can be solved by using Newton-Raphson, selecting some initial value condition. Depending on the initial value, Newton-Raphson will converge to the physical solution, the spurious one, or not converge at all. The interesting point is that the set of points for which the problem will converge to the physical solution is a fractal: we can find two values as close as desired having convergence to the physical solution for the first one and convergence to the spurious, or no convergence, for the other.

Additionally, if you increase the load, the set of points without convergence grows up until it covers all the possibilities except the solution at the voltage collapse point.

*Page 15--amend the fourth and fifth paragraphs*

[0077] In this simplistic example, the new method to be introduced behaves in an excellent manner with regard to approaching voltage collapse. For this problem, ~~all the area would always be colored white (indicating that every point would lead to the physical solution)~~ there is no need of initial condition, and the physical solution will always be found, including those situations very near to the voltage collapse point.

[0078] Extending the chaotic behavior of this two bus model to larger networks using traditional methods, it is clear that unreliable results can be introduced near voltage collapse for transmission grids.

*Page 16--amend the first and second full paragraphs*

[0080] 1. The physical solution must be connected in a continuous way to the non-load and non-generation case ~~[[0-case),]]~~ in which all the voltages are equal to the ~~nominal~~ normal or designed voltage level, and there is no energy flow in the links. The reason for this lies in the fact that the ~~[[0-case]]~~ no load case is physical (it is possible to build a real electrical power system with this state) and any other physical state can be reached by increasing simultaneously in a continuous way, load and generation until the final state is reached.

[0081] 2. The quantities that appear in the equations (voltages, power, and flows that are complex numbers) are constrained to have functional relations between them (Holomorphic function) with a very strong property called analyticity. This is a property of functions defined in the complex plane that reflects deeper symmetries of the system than is represented by the functions. In this case, analyticity is a property implied in the definition of the Ohm and Kirchov laws and, thus, by the load flow equations.

*Page 16--add a new paragraph after the second full paragraph (line 17)*

Holomorphic functions are the central object of study of complex analysis; they are functions defined on an open subset of the complex number plane  $\mathbb{C}$  with values in  $\mathbb{C}$  that are complex-differentiable at every point. This is a much stronger condition than real differentiability and implies that the function is infinitely often differentiable and can be described by its Taylor series (Power series expansion). The term analytic function is often used interchangeably with "holomorphic function", although note that the former term has several other meanings.

*Page 16--amend the fourth and fifth full paragraphs*

[0083] a. Embed the load flow problem  $L$  in a ~~parametric homotopy~~ holomorphic embedding  $L(s)$  that goes continuously from the ~~[[0-case ( $L(0)$ )]]~~ no load case ( $L(s=0)$ ) to the problem or objective case ~~[[ $(L(1))$ ]]~~  $(L(s=1))$ .

[0084] b. Develop, in power series, the values of the equation's unknowns in the parameters of the ~~homotopy~~ holomorphic embedding in a neighborhood of the ~~[[0-case]]~~ no load case value of the parameter

*Page 17--amend the fourth paragraph*

[0088] First, we construct ~~[[the]]~~ an embedding, defined as an extension of the function domain in one new variable, transforming the load flow equations into a function of a single complex variable.

*Page 17--amend the last paragraph that begins at line 22*

[0089] For an n-bus case, let  $Y_{ij}$  be the admittance matrix of an n-buses network (0 is a swing bus), with  $S_i$  and  $V_i$  the complex power and complex voltage at bus i. The ~~loadflow~~ load flow equations  $L(s)$  can be written as

*Page 18--amend the first paragraph and add a new paragraph before the last sentence*

[0090] In order to solve the load flow equation, we define an holomorphic embedding in a family of problems depending on a complex parameter  $s$  such that we know the solution for  $s=0$  (the no load case), and for  $s=1$  we recover the original equations.

As the equations are holomorphic, the knowledge of the power series expansion for a single value of  $s$ , determines in a unique way the values of the equations for all possible values  $s$  in the complex plane. Notice that in this case, knowledge of the power series expansion at one point is equivalent to knowledge of the full function for all  $s$  values. This is the main difference with the methodologies related to the homotopy continuation method, restricted to a real parameter, in which is necessary to follow a path in a predictor-corrector way by using only first order derivatives (not the full power series expansion).

One of the possible embeddings is:

*Page 19--amend the last paragraph*

[0105] Next, from the series coefficients, it is possible to build a ~~rational~~ n-order algebraic approximant for the function obtained by analytic continuation from the point  $s=0$  to  $s=1$ . There is a proof assuring that if the set of equations has a solution in the physical branch, it is always possible to find a continuation path from  $s=0$  (no charge) to  $s=1$ , free of singularities, and obtain the solution to the equation by evaluating the ~~rational function~~ algebraic approximant for  $s=1$ .

*Page 23--amend the fourth full paragraph*

[0116] The State Estimation process 1106 consists on standard least square minimization on the weighted differences and takes place using Gauss Seidel.